Image Enhancement: Frequency domain methods

- The concept of filtering is easier to visualize in the frequency domain. Therefore, enhancement of image $f(m,n)$ can be done in the frequency domain, based on its DFT $F(u,v)$.

- This is particularly useful, if the spatial extent of the point-spread sequence $h(m,n)$ is large. In this case, the convolution

PSS

$$\boxed{g(m,n) = h(m,n) * f(m,n)}$$

Enhanced Image Given Image

may be computationally unattractive.

- We can therefore directly design a transfer function $H(u,v)$ and implement the enhancement in the frequency domain as follows:

Transfer function

$$\boxed{G(u,v) = H(u,v)F(u,v)}$$

Enhanced Image Given Image

Lowpass filtering

- Edges and sharp transitions in grayvalues in an image contribute significantly to high-frequency content of its Fourier transform.

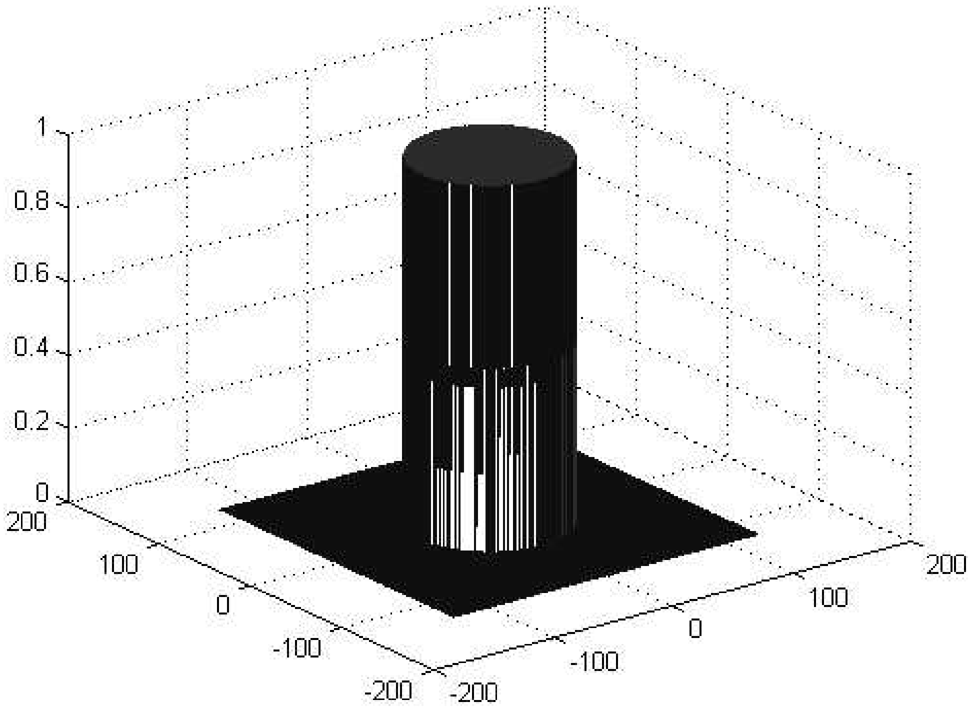
- Regions of relatively uniform grayvalues in an image contribute to low-frequency content of its Fourier transform.

- Hence, an image can be smoothed in the Frequency domain by attenuating the high-frequency content of its Fourier transform. This would be a lowpass filter!

- For simplicity, we will consider only those filters that are real and radially symmetric.

- An **ideal lowpass filter** with **cutoff frequency** r_0:

$$H(u,v) = \begin{cases} 1, & \text{if } \sqrt{u^2 + v^2} \le r_0 \\ 0, & \text{if } \sqrt{u^2 + v^2} > r_0 \end{cases}$$



Ideal LPF with $r_0 = 57$

- Note that the origin (0, 0) is at the center and not the corner of the image (recall the "`fftshift`" operation).

- The abrupt transition from 1 to 0 of the transfer function $H(u,v)$ cannot be realized in practice, using electronic components. However, it can be simulated on a computer.

Ideal LPF examples

Original Image

LPF image, $r_0 = 57$

LPF image, $r_0 = 36$

LPF image, $r_0 = 26$

- Notice the severe **ringing** effect in the blurred images, which is a characteristic of ideal filters. It is due to the discontinuity in the filter transfer function.

Choice of cutoff frequency in ideal LPF

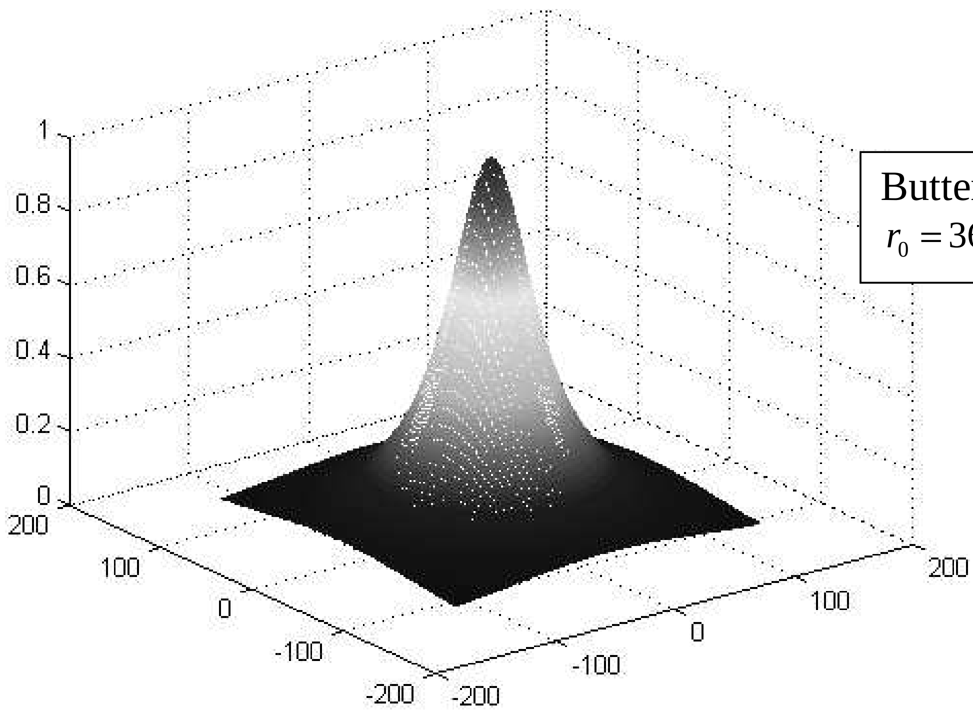
- The cutoff frequency r_0 of the ideal LPF determines the amount of frequency components passed by the filter.
- Smaller the value of r_0, more the number of image components eliminated by the filter.
- In general, the value of r_0 is chosen such that most components of interest are passed through, while most components not of interest are eliminated.
- Usually, this is a set of conflicting requirements. We will see some details of this is image restoration
- A useful way to establish a set of standard cut-off frequencies is to compute circles which enclose a specified fraction of the total image power.
- Suppose $P_T = \sum_{v=0}^{N-1} \sum_{u=0}^{M-1} P(u,v)$, where $P(u,v) = |F(u,v)|^2$, is the total image power.
- Consider a circle of radius $r_0(\alpha)$ as a cutoff frequency with respect to a threshold α such that $\sum_v \sum_u P(u,v) = \alpha P_T$.
- We can then fix a threshold α and obtain an appropriate cutoff frequency $r_0(\alpha)$.

Butterworth lowpass filter

- A two-dimensional Butterworth lowpass filter has transfer function:

$$H(u,v) = \frac{1}{1 + \left[\frac{\sqrt{u^2 + v^2}}{r_0} \right]^{2n}}$$

  - n: filter order, r_0: cutoff frequency



Butterworth LPF with $r_0 = 36$ and $n = 1$

- Frequency response does not have a sharp transition as in the ideal LPF.

- This is more appropriate for image smoothing than the ideal LPF, since this not introduce ringing.

Butterworth LPF example

Original Image

LPF image, $r_0 = 18$

LPF image, $r_0 = 13$

LPF image, $r_0 = 10$

Butterworth LPF example: False contouring

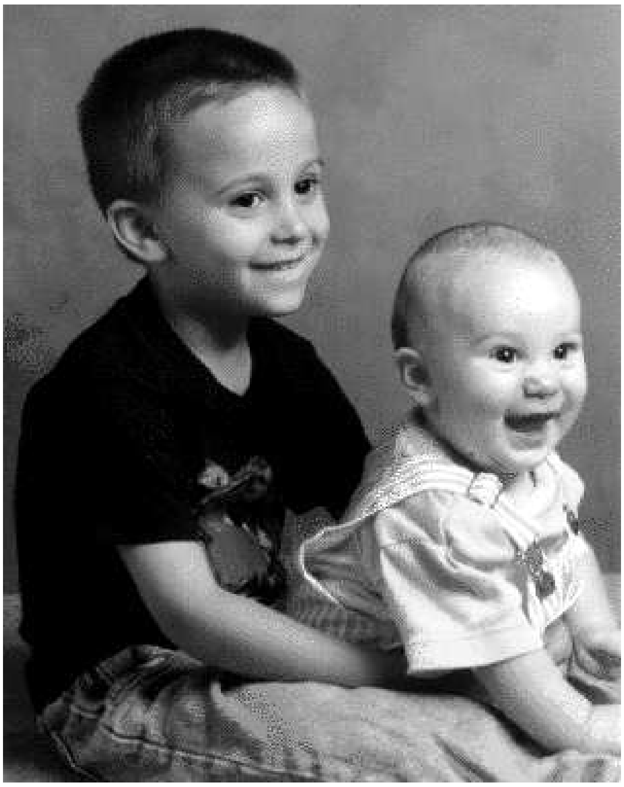
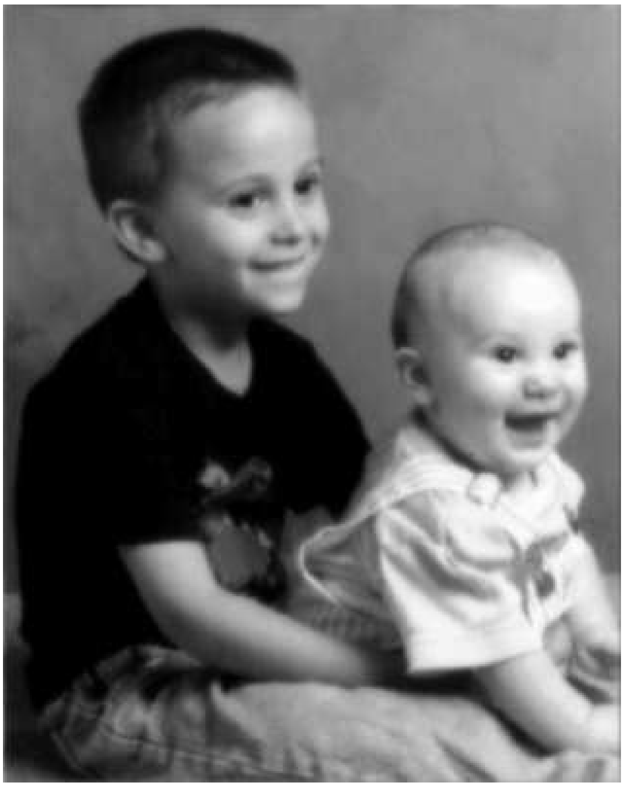
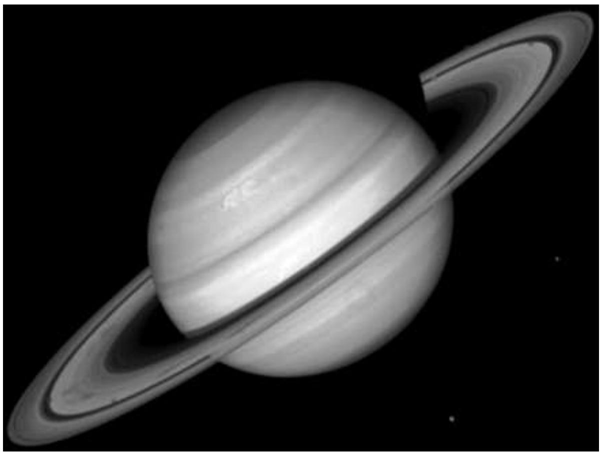


Image with false contouring due to insufficient bits used for quantization

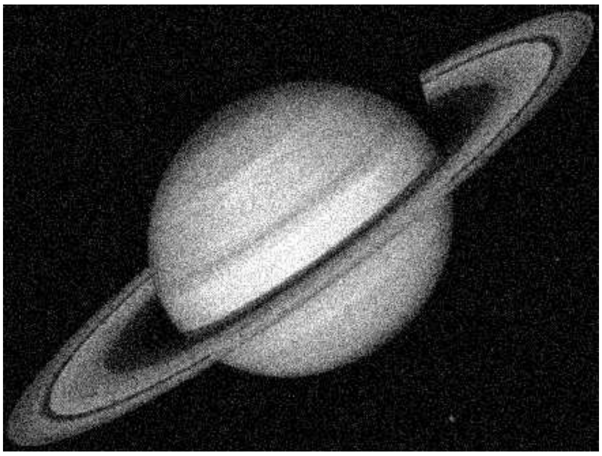


Lowpass filtered version of previous image

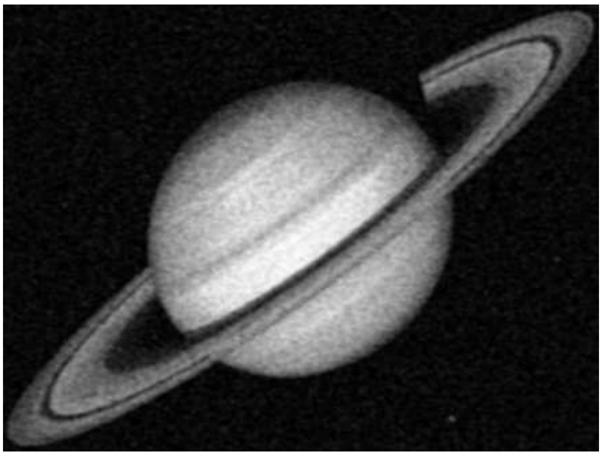
Butterworth LPF example: Noise filtering



Original Image



Noisy Image



LPF Image

Gaussian Low pass filters

- The form of a Gaussian lowpass filter in two-dimensions is given by $H(u,v) = e^{-D^2(u,v)/2\sigma^2}$, where $D(u,v) = \sqrt{u^2 + v^2}$ is the distance from the origin in the frequency plane.
- The parameter σ measures the spread or dispersion of the Gaussian curve. Larger the value of σ, larger the cutoff frequency and milder the filtering.
- When $D(u,v) = \sigma$, the filter is down to 0.607 of its maximum value of 1.
- See Example 4.6 in the text for an illustration.
- Also read section 4.3.4 for an application of lowpass filtering to text images.

Highpass filtering

- Edges and sharp transitions in grayvalues in an image contribute significantly to high-frequency content of its Fourier transform.

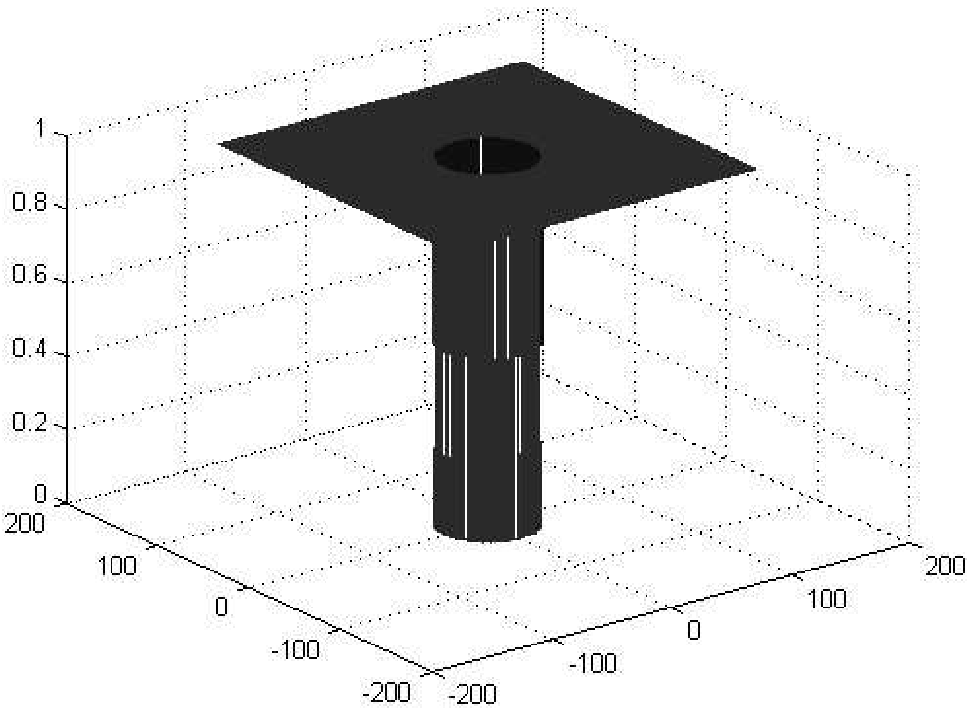
- Regions of relatively uniform grayvalues in an image contribute to low-frequency content of its Fourier transform.

- Hence, image sharpening in the Frequency domain can be done by attenuating the low-frequency content of its Fourier transform. This would be a highpass filter!

- For simplicity, we will consider only those filters that are real and radially symmetric.

- An **ideal highpass filter** with **cutoff frequency** r_0:

$$H(u,v) = \begin{cases} 0, \text{ if } \sqrt{u^2+v^2} \le r_0 \\ 1, \text{ if } \sqrt{u^2+v^2} > r_0 \end{cases}$$



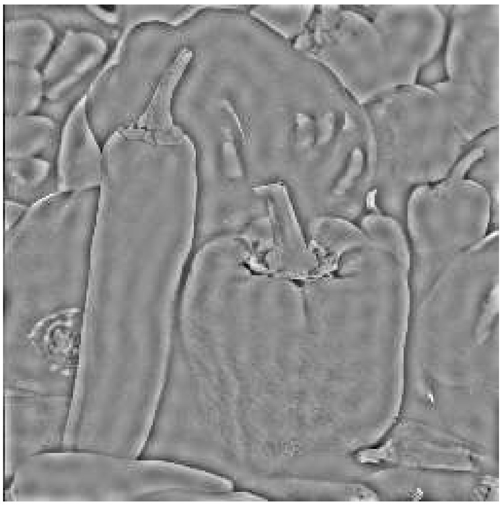
Ideal HPF with $r_0 = 36$

- Note that the origin (0, 0) is at the center and not the corner of the image (recall the "`fftshift`" operation).

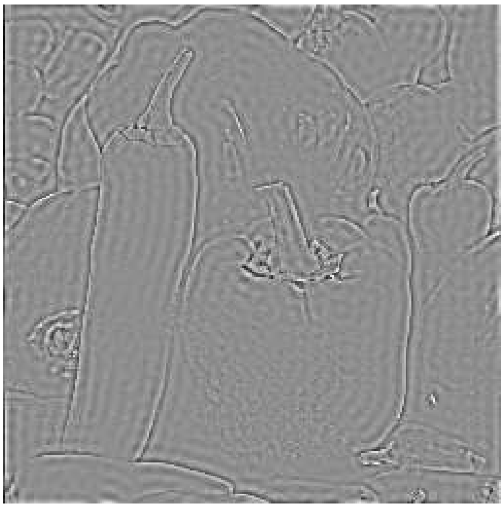
- The abrupt transition from 1 to 0 of the transfer function $H(u,v)$ cannot be realized in practice, using electronic components. However, it can be simulated on a computer.

Ideal HPF examples

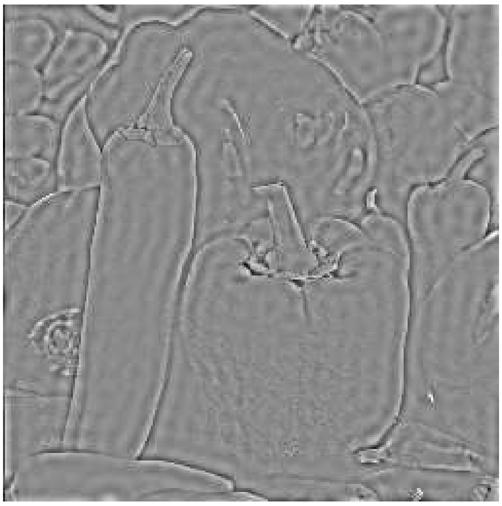
Original Image



HPF image, $r_0 = 18$



HPF image, $r_0 = 36$



HPF image, $r_0 = 26$

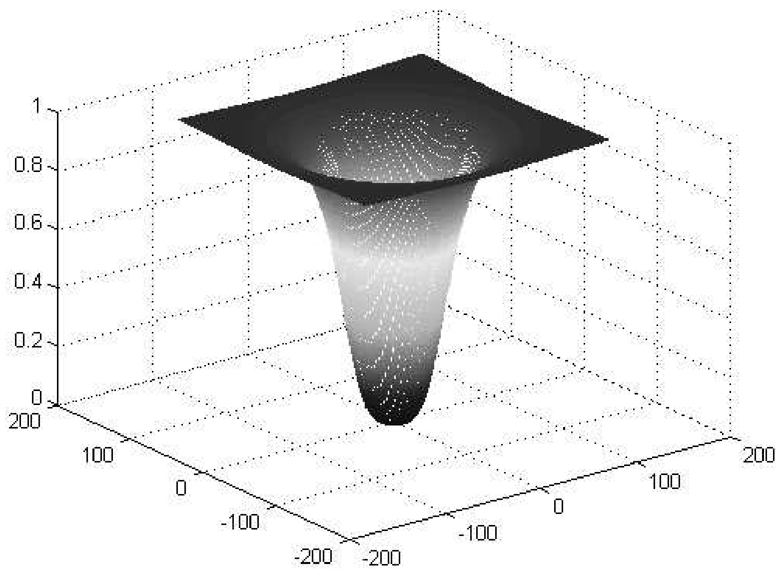
- Notice the severe **ringing** effect in the output images, which is a characteristic of ideal filters. It is due to the discontinuity in the filter transfer function.

Butterworth highpass filter

- A two-dimensional Butterworth highpass filter has transfer function:

$$H(u,v) = \frac{1}{1 + \left[\frac{r_0}{\sqrt{u^2 + v^2}} \right]^{2n}}$$

  - n: filter order, r_0: cutoff frequency



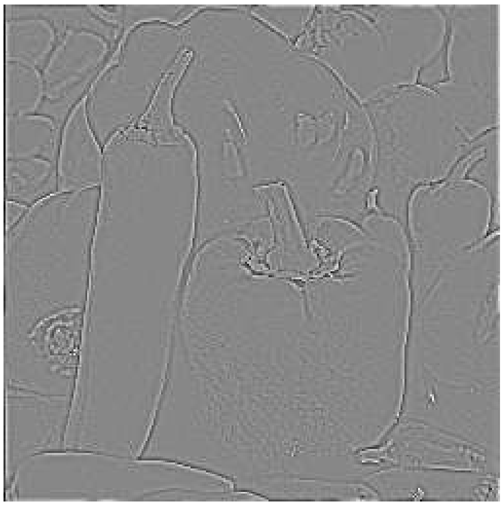
Butterworth HPF with $r_0 = 47$ and 2

- Frequency response does not have a sharp transition as in the ideal HPF.

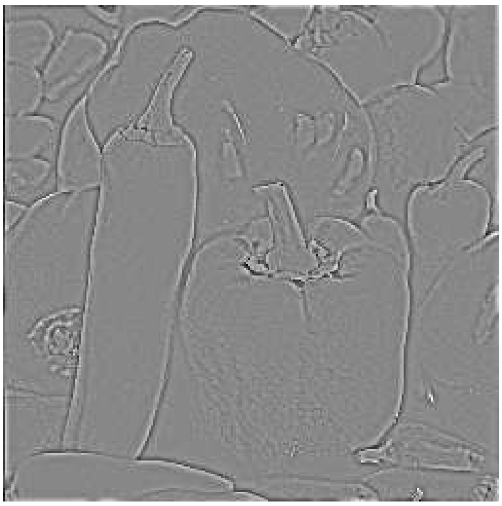
- This is more appropriate for image sharpening than the ideal HPF, since this not introduce ringing.

Butterworth HPF example

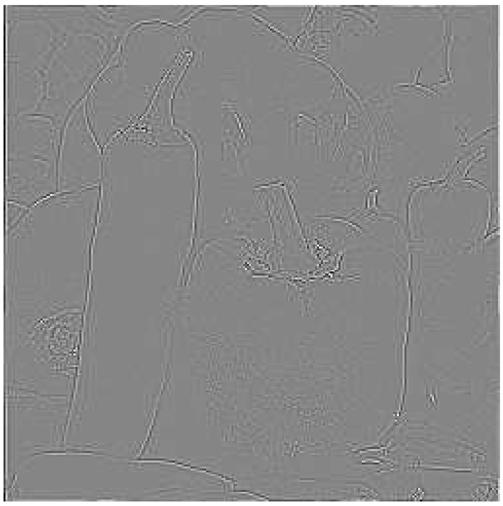
Original Image



HPF image, $r_0 = 47$



HPF image, $r_0 = 36$



HPF image, $r_0 = 81$

Gaussian High pass filters

- The form of a Gaussian lowpass filter in two-dimensions is given by $H(u,v) = 1 - e^{-D^2(u,v)/2\sigma^2}$, where $D(u,v) = \sqrt{u^2 + v^2}$ is the distance from the origin in the frequency plane.
- The parameter σ measures the spread or dispersion of the Gaussian curve. Larger the value of σ, larger the cutoff frequency and more severe the filtering.
- See Example in section 4.4.3 of text for an illustration.