Morphological Image Processing

Presented By:
Diwakar Yagyasen
Sr. Lecturer
CS&E, BBDNITM, Lucknow
Morphology

- About the **form** and **structure** of animals and plants

Mathematical morphology

- Using **set theory**
- Extract **image component**
- Representation and description of **region shape**
Sets in mathematical morphology represent objects in an image.

Example

- Binary image: the elements of a set is the coordinate \((x, y)\) of the pixels, in \(\mathbb{Z}^2\)
- Gray-level image: the element of a set is the triple, \((x, y, \text{gray-value})\), in \(\mathbb{Z}^3\)
Outline

- Preliminaries – set theory
- Dilation and erosion
- Opening and closing
- Hit-or-miss transformation
- Some basic morphological algorithms
- Extensions to gray-scale images
Preliminaries – set theory

- A be a set in $\mathbb{Z}^2$.

- $a = (a_1, a_2)$ is an element of $A$. $a \in A$

- $a$ is not an element of $A$ $a \notin A$

- Null (empty) set: $\emptyset$
Set theory (cont.)

- Explicit expression of a set
  1. $A = \{a_1, a_2, \ldots, a_n\}$
  2. $A = \{\text{element} \mid \text{condition for set elements}\}$

- Example:
  
  $C = \{w \mid w = -d, \text{ for } d \in D\}$
Set operations

- A is a **subset** of B: every element of A is an element of another set B
  \[ A \subseteq B \]

- Union
  \[ C = A \cup B \]

- Intersection
  \[ C = A \cap B \]

- Mutually exclusive
  \[ A \cap B = \emptyset \]
Graphical examples

\[ A \]

\[ B \]

\[ A \cup B \]

\[ A \cap B \]
Graphical examples (cont.)

\[ A^c = \{ w \mid w \notin A \} \]

\[ A - B = \{ w \mid w \in A, w \notin B \} \]
Logic operations on binary images

- Functionally complete operations
  - AND, OR, NOT

<table>
<thead>
<tr>
<th>p</th>
<th>q</th>
<th>p AND q (also p \cdot q)</th>
<th>p OR q (also p + q)</th>
<th>NOT (p) (also \overline{p})</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
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<td>0</td>
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<td>1</td>
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<td>0</td>
</tr>
</tbody>
</table>
Special set operations for morphology

**Translation**

\[(A)_z = \{c \mid c = a + z, \text{ for } a \in A\}\]

**Reflection**

\[\hat{B} = \{w \mid w = -b, \text{ for } b \in B\}\]
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Dilation

B: structuring element

\[ A \oplus B = \left\{ z \ | (\hat{B})_z \cap A \neq \emptyset \right\} \]
Dilation: another formulation

\[ A \oplus B = \left\{ z \left| (\hat{B})_z \cap A \subseteq A \right. \right\} \]
Application of dilation: bridging gaps in images

Effects: increase size, fill gap

Historically, certain computer programs were written using only two digits rather than four to define the applicable year. Accordingly, the company's software may recognize a date using "00" as 1900 rather than the year 2000.

max. gap = 2 pixels

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Erosion

\[ A \ominus B = \left\{ z \mid (B)_z \subseteq A \right\} \]

z: displacement

B: structuring element

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Erosion (cont.)

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Application of erosion: eliminate irrelevant detail

Squares of size 1, 3, 5, 7, 9, 15 pels

Erode with 13x13 square

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Dilation and erosion are duals

\[
(A \ominus B)^c = \left\{ z \mid (B)_z \subseteq A \right\}^c
\]

\[
= \left\{ z \mid (B)_z \cap A^c = \emptyset \right\}^c
\]

\[
A \oplus B = \left\{ z \mid (\hat{B})_z \cap A \neq \emptyset \right\}
\]

\[
= A^c \oplus \hat{B}
\]
Application: Boundary extraction

- Extract boundary of a set A:
  - First erode A (make A smaller)
  - \( A - \text{erode}(A) \)
Application: boundary extraction

original image

Using 5x5 structuring element

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Opening

- **Dilation**: expands image w.r.t structuring elements
- **Erosion**: shrink image
- erosion + dilation = original image?
- **Opening** = erosion + dilation

\[ A \circ B = (A \ominus B) \oplus B \]
Opening (cont.)

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Opening (cont.)

Smooth the contour of an image, breaks narrow isthmuses, eliminates thin protrusions.
**Closing**

- **Dilation + erosion = erosion + dilation?**
- **Closing = dilation + erosion**

\[ A \bullet B = (A \oplus B) \ominus B \]

---

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A \oplus B

A \cdot B = (A \oplus B) \ominus B
Closing (cont.)

Smooth the object contour, fuse narrow breaks and long thin gulfs, eliminate small holes, and fill in gaps
Properties of opening and closing

- Opening
  (i) $A \circ B$ is a subset (subimage) of $A$
  (ii) If $C$ is a subset of $D$, then $C \circ B$ is a subset of $D \circ B$
  (iii) $(A \circ B) \circ B = A \circ B$

- Closing
  (i) $A$ is a subset (subimage) of $A \bullet B$
  (ii) If $C$ is a subset of $D$, then $C \bullet B$ is a subset of $D \bullet B$
  (iii) $(A \bullet B) \bullet B = A \bullet B$
Noisy image

Remove outer noise

Remove inner noise

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Outline

- Preliminaries
- Dilation and erosion
- Opening and closing
- Hit-or-miss transformation
- Some basic morphological algorithms
- Extensions to gray-scale images
Hit-or-miss transformation

- Find the location of certain shape

\[(A \ominus X)\]

Find the set of pixels that contain shape \(X\)

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Hit-or-miss transformation

$A = X \cup Y \cup Z$

$W$

$(W - X)$

$A^c \ominus (W - X)$

Detect object via background

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Hit-or-miss transformation

- Eliminate unnecessary parts

\[(A \ominus X) \cap (A^c \ominus [W - X])\]

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Outline

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Basic morphological algorithms

- Extract image components that are useful in the representation and description of shape
- Boundary extraction
- Region filling
- Extract of connected components
- Convex hull
- Thinning
- Thickening
- Skeleton
- Pruning

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How?

Idea: place a point inside the region, then dilate that point iteratively.

\[ X_0 = p \]
\[ X_k = (X_{k-1} \oplus B) \cap A^c, k = 1,2,3,... \]

Until \[ X_k = X_{k-1} \]

Bound the growth

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Region filling (cont.)

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Application: region filling

Original image

The first filled region

Fill all regions

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Extraction of connected components

- Idea: start from a point in the connected component, and dilate it iteratively

\[
X_0 = p \\
X_k = (X_{k-1} \oplus B) \cap A, \ k = 1,2,3,... \\
\text{Until } X_k = X_{k-1}
\]

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Extraction of connected components (cont.)

\[ X_0 = p \]

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original

thresholding

erosion

<table>
<thead>
<tr>
<th>Connected component</th>
<th>No. of pixels in connected comp</th>
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<tbody>
<tr>
<td>01</td>
<td>11</td>
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<tr>
<td>02</td>
<td>9</td>
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<tr>
<td>15</td>
<td>85</td>
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</tbody>
</table>
How to define a Skeletons?

**Maximum disk**

1. The largest disk Centered at a pixel
2. Touch the boundary of A at two or more places

Recall: Balls of erosion!

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**Skeleton**

- **Idea: erosion**

<table>
<thead>
<tr>
<th>Erosions</th>
<th>Openings</th>
<th>Set differences</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>$A \circ B$</td>
<td>$A - (A \circ B)$</td>
</tr>
<tr>
<td>$A \ominus B$</td>
<td>$(A \ominus B) \circ B$</td>
<td>$(A \ominus B) - ((A \ominus B) \circ B)$</td>
</tr>
<tr>
<td>$A \ominus 2B$</td>
<td>$(A \ominus 2B) \circ B$</td>
<td>$(A \ominus 2B) - ((A \ominus 2B) \circ B)$</td>
</tr>
<tr>
<td>$A \ominus 3B$</td>
<td>$(A \ominus 3B) \circ B$</td>
<td>$(A \ominus 3B) - ((A \ominus 3B) \circ B)$</td>
</tr>
<tr>
<td>$\vdots$</td>
<td>$\vdots$</td>
<td>$\vdots$</td>
</tr>
<tr>
<td>$A \ominus kB$</td>
<td>$(A \ominus kB) \circ B$</td>
<td>$(A \ominus kB) - ((A \ominus kB) \circ B)$</td>
</tr>
</tbody>
</table>

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FIGURE 10.29 The final skeleton.

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Problem

- The scanned image is not adjusted well

  ![Diagram](image.png)

- How to detection the direction of lines?
- How to rotate?